CS 3343 Quiz 1

Student name ______________________ discussed with ______________________

Assume that $T(1)$ is in $\Theta(1)$. Solve the following recurrences using the recursion tree method.

(1) $T(n) = 4T(n/2) + n$
(2) $T(n) = 3T(n-2) + 1$

\[
T(n) = 4T\left(\frac{n}{2}\right) + n
\]

\[
\begin{align*}
\frac{n}{2} & \quad \frac{n}{2} \quad \frac{n}{2} \\
\frac{n}{4} & \quad \frac{n}{4} \quad \frac{n}{4} \\
\frac{n}{8} & \quad \frac{n}{8} \quad \frac{n}{8}
\end{align*}
\]

\[
h = \log_2 n
\]

\[
\sum_{i=0}^{h} n \cdot 2^i = \Theta(n^2) = \Theta(n \cdot n) = \Theta(n^2)
\]
$T(n) = 3T(n-2) + 1$

\[
\begin{align*}
   &1 \\
   &1 \\
   &1 \\
   &3 \\
   &3^2 \\
   &\vdots \\
\end{align*}
\]

\[
h = \frac{n}{2}
\]

\[
\sum_{i=0}^{h} 3^i = \Theta(3^h)
\]

\[
= \Theta(3^{\frac{n}{2}})
\]

\[
= \Theta((\sqrt{3})^n)
\]
CS 3343 Quiz 2

Student name 1 ___________________________  Student name 2 ___________________________

Assume that T(1) is in \( \Theta(1) \). Solve the following recurrence using the master method. If the master method cannot be applied, state the reason.

(a) \( T(n) = 3 \ T(n / 3) + n \)

\[
\frac{n^{\log_3 3}}{n} \quad \text{vs} \quad n
\]

\[
= n \quad \text{case 2}. \quad T(n) = \Theta(n \log n)
\]

(b) \( T(n) = 2 \ T(n / 2) + 1 \)

\[
\frac{n^{\log_2 2}}{1} \quad \text{vs} \quad 1
\]

\[
= n \quad \text{case 1}. \quad T(n) = \Theta(n)
\]

(c) \( T(n) = T(n / 5) + n \)

\[
\frac{n^{\log_5 1}}{1} \quad \text{vs} \quad n
\]

\[
= 1 \quad \text{case 3}. \quad T(n) = \Theta(n)
\]

(d) \( T(n) = 4T(n / 2) + n \log n \)

\[
\frac{n^{\log_2 4}}{n \log n} \quad \text{vs} \quad n \log n
\]

\[
= n^2 \quad n \geq n \log n \quad \text{for some} \quad \varepsilon.
\]

\[
\quad \text{case 1}. \quad T(n) = \Theta(n^2)
\]

(e) \( T(n) = 4T(n/4) + n \log n \)

\[
\frac{n^{\log_4 4}}{n \log n} \quad \text{vs} \quad n \log n
\]

\[
= n \quad \text{MM cannot be applied.}
\]

\[
\quad \text{case 2}. \quad T(n) = \Theta(n \log^2 n)
\]

(f) \( T(n) = 4T(n/4) + n / \log n \)

\[
\frac{n^{\log_4 4}}{n / \log n} \quad \text{vs} \quad n / \log n
\]

\[
= n \quad \text{MM cannot be applied.}
\]
CS 3343 Quiz 3

Student name _______________________  Discussed with _______________________

1. Define the recurrence for the running time of the following two algorithms.

(1) Alg1 (A[1..n])
   if (n < 4)
      return \( \sum \Theta(1) \) since \( n < 4 \)
   q1 = n / 4;
   q2 = 2 \times n / 4;
   q3 = 3 \times n / 4;

   if (n % 2 == 0)
      % recursive calls on first quarter and second half of A
      return Alg1(A[1..q1]) + Alg1(A[q2+1..n])
   else
      % recursive calls on first half and last quarter of A
      return Alg1(A[1..q2]) + Alg1(A[q3+1..n])
   end

   \( T(n) = T(n/4) + T(n/2) + \Theta(1) \)

(2) Alg2 (A[1..n])
   if (n < 4)
      return \( \sum \Theta(1) \)
   q1 = n / 4;
   q2 = 2 \times n / 4;
   q3 = 3 \times n / 4;

   if (n % 2 == 0)
      copy A[1..q1] (i.e., first quarter of A) into array B;
      copy A[q2+1..n] (i.e., second half of A) into array C;
   else
      copy A[q3+1..n] (i.e., last quarter of A) into array B;
      copy A[1..q2] (i.e., first half of A) into array C;
   end

   % recursive function calls on array B and array C.
   return Alg2(B) + Alg2(C);

   \( T(n) = T(n/4) + T(n/2) \)

\( T(n) = T(n/4) + T(n/2) + \Theta(n) \)

Continue on the back →
2. Use substitution method to prove that $T(n) = T(n/2) + T(n/4) + n \in O(n)$.

Proof (fill in the blanks):

According to the definition of $O$, we have to show that

$$T(n) \leq c \cdot n$$

for some $c > 0$ and all $n > n_0$.

Assume that this is true for $T(n/4)$ and for $T(n/2)$, which means

$$T(n/2) \leq c \cdot n/2$$

and

$$T(n/4) \leq c \cdot n/4$$

Substitute $T(n/4)$ and $T(n/2)$ in the recurrence by the corresponding right hand side of the above inequalities, we have

$$T(n) = T(n/2) + T(n/4) + n$$

$$\leq c \cdot n/2 + c \cdot n/4 + n$$

$$\leq c \cdot \frac{3}{4} n + n$$

$$\leq c \cdot n - \frac{1}{4} (c \cdot n + n)$$

$$\leq c \cdot n + (n - \frac{1}{4} (c \cdot n))$$

If $n - \frac{1}{4} (c \cdot n) \leq 0$

$$\Rightarrow n \leq \frac{c}{4} \cdot n$$

$$\Rightarrow 1 \leq \frac{c}{4} \cdot n \Rightarrow 4 \leq c \cdot n$$

Therefore, we have

$$T(n) \leq c \cdot n$$

for $c \geq 4$ and all $n > 0$,

which means $T(n) \in O(n)$ by definition of $O$. 