1. (30 points) Assume that $T(1) \in \Theta(1)$. Solve the following recurrences using the recursion tree method.
   a. $T(n) = 4T(n/2) + n^2$
   b. $T(n) = T(n - 2) + n$
   c. $T(n) = 4T(n/2) + n$
   d. $T(n) = 2T(n - 2) + 1$
   e. $T(n) = T(n/2) + T(n/3) + n$

2. (40 points) Assume that $T(1) \in \Theta(1)$. Solve the following recurrence functions using the master method. If the master method cannot be applied, state the reason, and give an upper bound (big-Oh) as tight as you can. Justify your answer.
   a. $T(n) = 8T(n/2) + n^2$
   b. $T(n) = T(3n/5) + n$
   c. $T(n) = 9T(n/3) + n^2$
   d. $T(n) = 16T(n/4) + n \log n$
   e. $T(n) = 2T(n/4) + \log^2 n$
   f. $T(n) = 3T(n/3) + \log n$
   g. $T(n) = 4T(n/4) + n \log n$
   h. $T(n) = 2T(n/4) + \sqrt{n}$
   i. $T(n) = 3T(n/3) + (n + \log n)$
   j. $T(n) = 2T(n/2) + n/\log n$

3. (Optional: 10 points) Assume that $T(1) \in \Theta(1)$. Solve the following recurrence functions using the master method and change of variables.
   a. $T(n) = 3T(n - 2) + n$
   b. $T(n) = T(n - 2) + n^3$
4. (20 points) Analysis of recursive algorithm. Consider the pseudocode of the following three algorithms for computing $2^n$, where $n$ is a non-negative integer.

```
Alg1 (n)
  if (n == 0) return 1;
  return 2 * Alg1(n - 1);
end

Alg2 (n)
  if (n == 0) return 1;
  return Alg2(n - 1) + Alg2(n - 1);
end

Alg3 (n)
  if (n == 0) return 1;
  m = floor (n / 2);
  p = Alg3(m);
  p = p * p;
  if (n % 2 == 1) // n is an odd number
    return 2 * p;
  else // n is an even number;
    return p;
  end
end
```

a. (3 points) Trace the three algorithms using two small examples ($n = 2$ and $n = 3$) to find out the outputs of the three algorithms, respectively.

b. (2 points) Briefly argue the correctness of Alg2 using induction.

c. (6 points) Let $A(n)$, $B(n)$, and $C(n)$ be the running time of the three algorithms, respectively, as a function of $n$. Write down the recurrence relations for $A(n)$, $B(n)$, and $C(n)$.

d. (9 points) Analyze and compare the running time of the three algorithms above. You can use either recursion tree of master method to solve the recurrences.

5. (10 points) Assume that $T(1) \in \Theta(1)$ and $T(n) = T(3n/4) + T(n/2) + n^2$. Prove $T(n) \in \Theta(n^2)$ using the substitution method.

6. **Bonus** (5 points) How much time did you spend on this homework? Who did you discuss with and what was the discussion about? What do you think about the difficulty level of the homework (harder than expected? just all right? easy?) What is the most difficult part? Do you have any comments/suggestions about the lecture, recitation, and homework?