3. (30 points) Heap and heap sort.
   a. (5 points) Is the sequence \[23, 17, 14, 6, 13, 10, 1, 5, 7, 12\] a max-heap?
   b. (5 points) What are the minimum and maximum numbers of elements in a heap of height \(h\)? Briefly justify your answer.
   c. (5 points) Study the pseudocode of Heapify and BuildHeap on Slide #16 and #29 in lecture10.ppt. Using Slides #30 – 42 as a model, illustrate the operation of BuildHeap on array \(A = [14 15 10 5 6 16 11 3 12 20]\). Figure 1 on page 3 is provided for your convenience. You only need to show the content of the tree after each call to heapify(). Make sure your final tree is indeed a heap.
   d. (5 points) Starting from the heap shown in Figure 2 (page 4), show the content of the new heap after each heap operation.
   e. (5 points) Why do we want the loop index \(i\) in algorithm BuildHeap to decrease from \(\lfloor \text{length}(A)/2\rfloor\) to 1 rather than increase from 1 to \(\lfloor \text{length}(A)/2\rfloor\)?
   f. (5 points) What is the running time of heapsort on an array \(A\) of length \(n\) that is already sorted in increasing order? What about decreasing order? (Hint: first think about the cost for building heap in these two cases, then think about the cost for the actual sorting part. You can use examples \([0 1 2 3 4 5 6 7 8 9]\) or \([9 8 7 6 5 4 3 2 1 0]\) to help you think.)

4. (15 points) Building a heap using insertion.
   The procedure BuildHeap can also be implemented by repeatedly using HeapInsert to insert the elements into the heap. Consider the following implementation:

   ```
   BuildHeap2(A)
   heapsize(A) = 1;
   for (i = 2 to length(A))
       HeapInsert(A, A[i]);
   ```

   a. (5 points) Compare the procedure BuildHeap2 with the procedure BuildHeap on Slide #29 in lecture10.ppt. Do the two procedures always create the same heap when run on the same input array? Prove that they do, or provide a counterexample.
   b. (5 points) What is the worst-case time complexity of BuildHeap2? Briefly justify your answer.
   c. (5 points) How does the running time of the two procedures compare?
Figure 1: Build heap
Figure 2: Heap operations